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THERMO-ELASTIC-PLASTIC STRESSES IN HOLLOW CYLINDERS DUE TO QUENCHING

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A hollow cylindrical tube rapidly quenched for the purpose of developing a high strength material structure is analyzed. The quenching creates severe thermal stresses early in the quenching cycle while later the material transformation by virtue of a volume change in the transformed material causes large transformation stresses. The transient temperature distributions and the elastic treatment of the stresses has been treated previously. The present work is an attempt to consider the thermo-elastic-plastic aspects of the

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problem. The von Mises' yield criterion and the Prandtl-Reuss stress strain relations are used. Results are calculated based on a new finite difference approach.

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SUMMARY. A hollow cylindrical tube, rapidly quenched for the purpose of developing a high strength material structure, is analyzed. The quenching creates severe thermal stresses early in the quenching cycle while later the material transformation by virtue of a volume change in the transformed material, causes large transformation stresses. The transient temperature distributions and the elastic treatment of the stresses has been treated previously. The present work is an attempt to consider the thermo-elastic-plastic aspects of the problem. The von Mises yield criterion and the Prandtl-Reuss stress strain relations are used. Results are calculated based on a new finite difference approach.

I. INTRODUCTION. Watervliet Arsenal has recently been developing techniques for the production of large caliber weapons using a rotary forge. Force hammers, evenly spaced at 90° intervals, strike the outside diameter of a hot (1500°F-1600°F), hollow cylindrical preform at the rate of 200 blows/minute. The final outside tube profile is programmed into the forge itself, and the wall thickness of the tube is varied as preprogrammed. The inside diameter of the tube is maintained constant by a mandrel which is water cooled. After the tube has been formed, it is allowed to cool to room temperature.

Once formed, the tube must then be heat treated. This procedure begins by heating the tube to 1650°F in an austenitizing furnace so that the austenite phase is developed throughout the material. The tube is then rapidly quenched so that the desired martensite phase is developed. This quenching is accomplished by spraying a large volume of water on both the inside and outside diameters. The tube is finally put through a tempering furnace at 1200°F.

Interest in the analytical studies of the process first arose when cracks began developing in the tube during quenching. While the possible causes of quench-cracking are many, most often they are associated with the material used. It was also decided, however, to look into the transient temperatures during the quenching process and the thermal and transformation stresses involved. The transformation stresses occur mainly due to volume changes in the material as it transforms from one

phase to another. As they are due to volume changes, transformation stresses can be treated in a manner similar to the thermal stresses. Although quench-cracking cannot be predicted from a study such as this, a better understanding of the quenching procedure will emerge and the relative severity of different quenching procedures would be known.

The transient temperatures and the zones of transformed material assuming a linear relationship for the change in volume between the martensite start and finish temperatures were treated in [1]. This reference also considers the thermal and transformation stresses assuming the stresses remain elastic. The present work seeks to incorporate an elastic-plastic stress analysis into the problem. In view of the previous results, this assumption is more realistic. The temperature and stress problem are considered uncoupled.

II. PROBLEM DESCRIPTION. The problem being considered is that of the elastic-plastic stresses developed during the quenching process. These stresses are due to both the transient temperatures that exist and the transformation stresses.

Most of the elastic-plastic analysis work on thick-wall cylinders concerns itself with mechanical loadings. Bland [2] does consider thermal loads on a thick wall tube. Tresca's yield criterion and its associated flow rule wereused to obtain solutions to tubes of work-hardening material subjected to both internal and external pressures. The temperatures, however, are steady state,and the thermal stresses due to this steady state temperature distribution are first calculated and assumed elastic. External or internal pressures are then applied until some desired plastic state is arrived at. S. C. Chu [3] used the incremental approach for solving the problem of elastic-plastic thick-walled tubes subject to transient thermal loadings. The von Mises yield criterion and Prandtl-Reuss equations are used.

In the area of elastic-plastic analysis for transformation stresses, the bulk of the work comes from a series of papers by Zwicky, Landau, Weiner, and Huddleston [4-6]. Of those that consider the cylinder configuration, Weiner and Huddleston [5] used the Tresca yield criterion and the associated flow rule to compute the residual stresses in the cylinders. The problem for the transformation stresses was solved by assuming that the volume expansion of the transformed material was equivalent to that of a temperature discontinuity progressing inward from the surface. They considered a solid cylinder of incompressible material. Landau and Zwicky [6] solved a similar problem using the von Mises yield criterion and its associated flow rule. They assumed a compressible material, the yield point stress to be a function of temperature, and included the computation of transient thermal stresses.

The problem considered here is that of determining the thermo-elastic-plastic stresses and transformation stresses in a cylinder due to quenching. The thermal program developed in [1] was coupled to a program [7] for the computation of elastic-plastic stresses in a thick-walled cylinder subjected to internal and external pressure. The problem is assumed to be axisymmetric.

The computer program for the temperature distribution allows for a transient analysis with temperature dependent material properties using an implicit finite difference scheme. The computer program for the elastic-plastic stresses uses an incremental approach. It has been altered to include stresses due to thermal loads. The von Mises yield criterion is used with the associated Prandtl-Reuss flow rule. The material is assumed compressible and is capable of work-hardening although for this work the material was assumed to be elastic-perfectly plastic.

III. THERMAL EQUATIONS. The partial differential equation for the temperature (T) in a thick-wall cylinder with inner radius, a, and outer radius, b, is given in dimensionless form by

$$\frac{1}{r}\frac{\partial}{\partial r}\left[k(T)r\frac{\partial T}{\partial r}\right] = c(T)\rho(T)\frac{\partial T}{\partial t} \tag{1}$$

where r is dimensionless radial distance, k(T), c(T), ρ(T) are dimensionless thermal conductivity, specific and density, respectively, and t is dimensionless time. The dimensionless quantities are defined as

$$r = \frac{\bar{r}}{b}, \quad T = \frac{\bar{T}-T_o}{T_i-T_o}$$

$$t = \frac{k_o}{\rho_o c_o b^2}\bar{t} \tag{2}$$

$$k(T) = k_o K(T), \quad c(T) = c_o C(T), \quad \rho(T) = \rho_o R(T)$$

and $\bar{r}$ is the radius, $\bar{T}$ is the temperature, k_o, c_o, ρ_o are thermal conductivity, specific heat and density at reference ambient temperature T_o; T_i is initial temperature; and $\bar{t}$ is time.

The boundary conditions are written as

$$\frac{\partial T}{\partial r} - h_1 T = -g_1 \quad \text{at } r = a/b$$

and

$$\frac{\partial T}{\partial r} - h_2 T = -g_2 \quad \text{at } r=1 .$$

With the boundary conditions expressed in this manner, different conditions at the boundary can be specified. If, e.g., $g_1 = 0$ and $h_1 \neq 0$ and finite, then a convection type boundary condition exists on the inner surface. If h_1 was very large and $g_1 = 0$, then $T = 0$ is specified. If h_2 and g_2 are both large and not equal, then the temperature $T = g_2/h_2$ is specified in the outer surface.

IV. STRESS EQUATIONS. The use of finite difference equations to solve the thermo-elastic-plastic stress problem requires expressing the equilibrium equation and the equation of compatibility at each node at which the finite difference equations are desired. The Prandtl-Reuss flow rule is used to eliminate the incremental stresses so that what results is a matrix for evaluating the incremental radial and tangential strains at each node. The required equations follow, written in dimensionless form. The problem is treated as plane strain.

The equation of equilibrium is written

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{3}$$

where

$$\sigma_r (= \frac{\bar{\sigma}_r}{\sigma_o}) \text{ is the dimensionless radial stress}$$

$$\sigma_\theta (= \frac{\bar{\sigma}_\theta}{\sigma_o}) \text{ is the dimensionless tangential stress}$$

and σ_o is the yield stress in tension, and the compatibility equation

$$\frac{\partial \varepsilon_\theta}{\partial r} + \frac{\varepsilon_\theta - \varepsilon_r}{r} = 0 \tag{4}$$

where

$$\varepsilon_\theta (= E\,\frac{\bar{\varepsilon}_\theta}{\sigma_o}) \text{ is dimensionless tangential strain}$$

$$\varepsilon_r (= E\,\frac{\bar{\varepsilon}_r}{\sigma_o}) \text{ is dimensionless radial strain}$$

and E/σ_o is yield strain in tension when E is Young's Modulus. The compressibility of the material is expressed by

$$\varepsilon = \alpha T + \frac{\sigma}{3K} \tag{5}$$

where

$$\varepsilon = \frac{1}{3}(\varepsilon_r + \varepsilon_\theta) \text{ is mean strain}$$

$$\sigma = \frac{1}{3}(\sigma_r + \sigma_\theta + \sigma_z) \text{ is mean stress}$$

$$K (= \frac{\bar{K}}{\sigma_o}) \text{ is dimensionless bulk modulus}$$

$\alpha(=\alpha T_i)$ is dimensionless coefficient of thermal expansion

and

$$\varepsilon_z = 0 \text{ for plane strain.}$$

Traction free boundary conditions are used

$$\sigma_r = 0 \text{ at } r = a/b \text{ and } r = 1. \tag{6}$$

It was desirable to write the finite difference equations in terms of strain alone, hence, the stresses in the equations of equilibrium had to be expressed in terms of the strains. This was accomplished by modifying a plastic stress-strain matrix [8] which was derived by inverting the Prandtl-Reuss equations. The inverted Prandtl-Reuss equation is

$$\{d\sigma\} = [D^P]\{d\varepsilon\} - \frac{E\alpha dT}{(1-2\nu)\sigma_o}\{1\} \tag{7}$$

where the stress vector is $\{d\sigma\} = \{d\sigma_r, d\sigma_\theta, d\sigma_z\}^T$, the strain vector $\{d\varepsilon\} = \{d\varepsilon_r, d\varepsilon_\theta, 0\}^T$, and $\{1\}$ represents a unit vector. The plastic stress-strain matrix $[D^P]$ is given by

$$[D^P] = \frac{1}{1+\nu}\begin{bmatrix} \frac{1-\nu}{1-2\nu} - \frac{\sigma_r'^2}{S} & & \text{SYMMETRIC} \\ \frac{\nu}{1-2\nu} - \frac{\sigma_r'\sigma_\theta'}{S} & \frac{1-\nu}{1-2\nu} - \frac{\sigma_\theta'^2}{S} & \\ \frac{\nu}{1-2\nu} - \frac{\sigma_r'\sigma_z'}{\ } & \frac{\nu}{1-2\nu} - \frac{\sigma_\theta'\sigma_z'}{S} & \frac{1-\nu}{1-2\nu} - \frac{\sigma_z'^2}{S} \end{bmatrix} \tag{8}$$

The primed stresses are deviatoric stresses,

$$\sigma_i' = \sigma_i - \frac{1}{3}\sigma \qquad i = r,\theta,z. \tag{9}$$

At each node during a computation, the von Mises yield criterion

$$\frac{1}{2}\left[(\sigma_r-\sigma_\theta)^2 + (\sigma_\theta-\sigma_z)^2 + (\sigma_z-\sigma_r)^2\right] = 1 \tag{10}$$

is checked to see if plastic deformation has progressed to that node. If not, the stresses remain elastic and can still be computed using (8) by setting the deviatoric stresses equal to zero. The matrix $[D^P]$ then becomes the same matrix as would exist if linear elastic behavior had been assumed. The quantity S is given by

$$S = \frac{2}{3}\bar{\sigma}^2\left(1 + \frac{H'}{3G}\right) \tag{11}$$

where

$$\bar{\sigma} = \frac{3}{2}\sigma_{ij}'\sigma_{ij}' = \frac{3}{2}(\sigma_r'^2 \;\; \sigma_\theta'^2 \;\; \sigma_z'^2) \tag{12}$$

is the equivalent stress and

$$H' = \frac{d\bar{\sigma}}{d\bar{\varepsilon}_p} \tag{13}$$

is the slope of the equivalent stress/equivalent plastic strain curve and is a measure of hardening. The increment in equivalent plastic strain is given by

$$d\bar{\varepsilon}_p = \frac{2}{3}\, d\varepsilon_{ij}^p d\varepsilon_{ij}^p \; . \tag{14}$$

V. NUMERICAL COMPUTATIONS. The Crank-Nicolson representation for finite differences of the partial differential equation governing the temperatures in time is [1]

$$\begin{aligned}
&[(a+i\Delta r)k_{i+\frac{1}{2},n+\frac{1}{2}}]T_{i+1,n+1} + \\
&+[-(a+i\Delta r)k_{i+\frac{1}{2},n+\frac{1}{2}}-(a+(i-1)\Delta r)k_{i-\frac{1}{2},n+\frac{1}{2}}-c_{i,n+\frac{1}{2}}p_{i,n+\frac{1}{2}}(\frac{2\Delta r^2}{\Delta t})(a+(i-\tfrac{1}{2})\Delta r)]T_{i,n+1} \\
&+ [(a+(i-1)\Delta r)k_{i-\frac{1}{2},n+\frac{1}{2}}]T_{i-1,n+1} = [-(a+i\Delta r)k_{i+\frac{1}{2},n+\frac{1}{2}}]T_{i+1,n} + \\
&+ [(a+i\Delta r)k_{i+\frac{1}{2},n+\frac{1}{2}}+(a+(i-1)\Delta r)k_{i-\frac{1}{2},n+\frac{1}{2}}-c_{i,n+\frac{1}{2}}p_{i,n+\frac{1}{2}}(\frac{z\Delta r^2}{\Delta t})(a+(i-\tfrac{1}{2})\Delta r)]T_{i,n} \\
&+ [-(a+(i-1)\Delta r)k_{i-\frac{1}{2},n+\frac{1}{2}}]T_{i-1,n} \; .
\end{aligned} \tag{15}$$

The equation is solved twice:

1. At $n+\frac{1}{2}$ step, allowing k,p,c etc. to take on the values at t=n step.

2. The new temperatures are then used to evaluate k,c,p, at $n+\frac{1}{2}$ step and the set of equations re-evaluated for the temperatures at the n+1 step.

The computed temperature distributions at each full time step are saved on disk and eventually called in when required by the stress program.

The finite difference equations are (for solid cylinder).
Compatibility:

$$-r_i \Delta\varepsilon_{\theta_{i-1}} + (2r_i - r_{i-1})\Delta\varepsilon_{\theta_i} - (r_i - r_{i-1})\Delta\varepsilon_{r_i} =$$

$$-r_i(\varepsilon_{\theta_i} - \varepsilon_{\theta_{i-1}}) - (r_i - r_{i-1})(\varepsilon_{\theta_i} - \varepsilon_{r_i}) \tag{16}$$

Equilibrium:

$$-r_i \Delta\sigma_{r_{i-1}} - (r_i - r_{i-1})\Delta\sigma_{\theta_i} + (2r_i - r_{i-1})\Delta\sigma_{r_i} =$$

$$-r_i(\sigma_{r_i} - \sigma_{r_{i-1}}) - (r_i - r_{i-1})(\sigma_{r_i} - \sigma_{\theta_i}) \tag{17}$$

Substituting the Prandtl-Reuss equations into that of equilibrium

$$-r_i D(r,\theta)\Delta\varepsilon_{\theta_{i-1}} - r_i D(r,r)\Delta\varepsilon_{r_{i-1}} + [-(r_i - r_{i-1})D(\theta,\theta) + (2r_i - r_{i-1})D(r,\theta)]\Delta\varepsilon_{\theta_i}$$

$$+ [-(r_i - r_{i-1})D(\theta,r) + (2r_i - r_{i-1})D(r,r)]\Delta\varepsilon_{r_i}$$

$$r_i[\sigma_{r_{i-1}} - \sigma_{r_i}] + (r_i - r_{i-1})(\sigma_{\theta_i} - \sigma_{r_i}) + r_i \frac{E\alpha}{1-2\nu}[\Delta T_i - \Delta T_{i-1}] \tag{18}$$

at i = 1 (zero radius for solid cylinder)

$$-\Delta\varepsilon_{\theta_1} + \Delta\varepsilon_{r_1} = \varepsilon_{\theta_1} - \varepsilon_{r_1} \tag{19}$$

at i = n (or outside boundary) $\sigma_r = 0$ or

$$D(r,\theta)\Delta\varepsilon_{\theta_n} + D(r,r)\Delta\varepsilon_{r_n} = \frac{E\alpha\Delta T_n}{1-2\nu} \quad . \tag{20}$$

For the hollow cylinder, a boundary condition similar to i = n can be written for i = 1.

The solution procedure for the transient temperature problem is as follows. The temperature problem is solved, and the temperature distributions at their computation times are stored on disk. These distributions are called into the thermo-elastic-plastic stress program one at a time. The corresponding thermal stresses are calculated and each

node checked to see if the yield criterion is satisfied. If not, the problem is still assumed to be elastic, a new temperature distribution is called in, and new stress increments calculated. The stresses are updated, and the yield criterion checked again. When the stresses at a point are found to satisfy the yield criterion the node is identified, and the stress increments at that node from the next set of temperatures are computed using the Prandtl-Reuss equation or $[D^P]$ matrix identified earlier. This procedure is continued with new sets of temperature called in and with the tracking of the elastic-plastic boundary(s) with time. The resultant stresses that exist after a steady-state or uniform temperature distribution is reached are the residual stresses.

The solution procedure for the transformation stresses is similar and will be described in the next section.

VI. RESULTS AND DISCUSSION. Several runs were made for the stresses due to the transient temperatures and for the transformation in both solid and hollow cylinders. For the results presented here, the following data were used:

$E = 30 \times 10^6$ psi, $\sigma_o = 30 \times 10^3$ psi

$\bar{\alpha} = 7.75 \times 10^{-6}/°F$, $\bar{T}_i$ - $1250°F \rightarrow \alpha \quad \bar{\alpha}\bar{T}_i = .0097$

$\nu = .3$

$h_2 = 12.2$, $h_1 = 12.2$ and 6.1

The first results shown are those for the transformation stresses. These stresses can be computed using the thermal stress formulation if one replaces αT or the thermal expansion by the linear expansion of the transformation. If the material expansion due to the transformation is isotropic, this linear change is 1.3 the volume expansion. As an example, the volume expansion in going from the austenite to the martensite structure for steel is about 3%-4%. In the quenching of a solid cylinder, the transformation begins on the outer surface and progresses inward to the center. Figure 1 shows the residual stresses in a solid cylinder due to a transformation occurring in the material. The insert shows the temperature function as it progresses inward. It is of unit height in the transformed material and zero in the untransformed material. The transformation is assumed to be occurring over eight nodes or 8% of the cylinder (indicated by N in Figure), and a linear relation is assumed over this length. Initially, σ_θ is compressive near the outside radius when the transformation just begins as the material wants to expand but is prevented from doing so by the surrounding untransformed material. As the transformation progresses, however, σ_θ slowly changes sign and becomes tensile. The computer run was stopped just before the transformation was complete, and that is the reason for

the behavior of the stresses near the bore. The transformation at r = 0 had just started when the run was stopped.

Figure 2 shows similar results for the hollow cylinder. The assumption is made that the transformation progressed evenly from the inside and outside surfaces. Tangential stresses on the outside surface again were initially compressive and slowly changed to tensile stresses while those in the inside radius always remained compressive.

Figures 3 and 4 show the residual stresses that exist due to the transient temperatures from the quenching process. Figure 3 represents the results for the solid cylinder. The large axial stress due to the plane strain assumption is easily seen. As the quenching begins, the outside surface cools and wants to contract. It is prevented from doing so by the surrounding material and therefore σ_θ is initially a tensile stress. The elastic-plastic boundary begins on the outside surface of the cylinder and moves toward the center.

Figure 4 shows similar results for the hollow cylinder. The figure shows the residual stresses when equal convection type boundary conditions are used on both the inside and outside diameters. These are shown by the solid lines. A comparison is made with the same problem when the convection boundary condition on the inside diameter is decreased by 50%. A dotted line compares the differences in the tangential stress, σ_θ, and a substantial reduction is noted in the residual stress.

The usefulness of these results is thus shown. For the quenching problem, the maximum quench time for the desired metallurgical phase structure to be formed is of interest because it implies that the material will be subjected to slower transient temperatures and smaller residual stresses. Thus, a better understanding of the transient temperatures in the quench tube, the resulting residual stresses, and the effect of the quenching process is gained.

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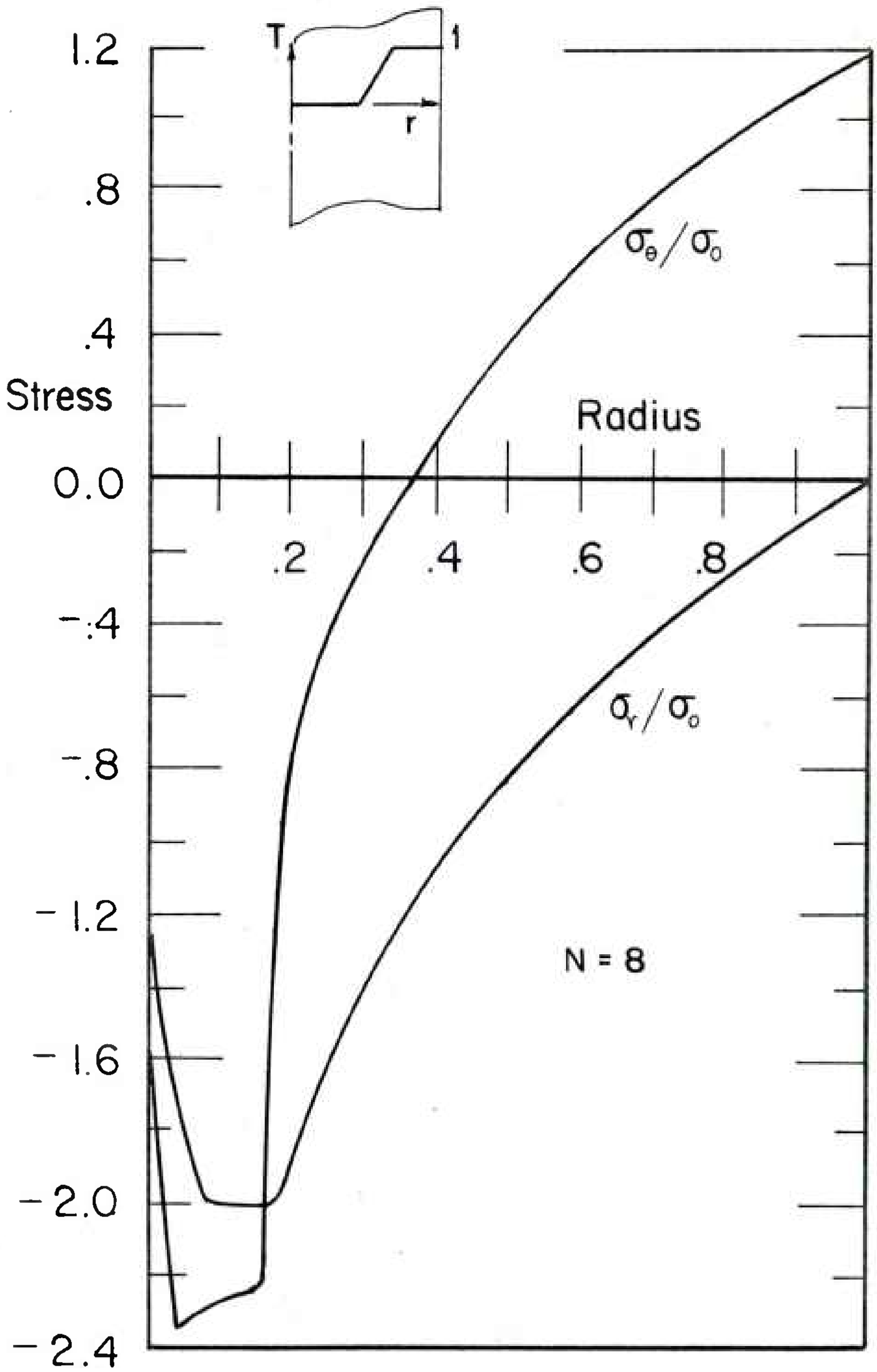

Figure 1. Residual Stresses in a Solid Cylinder Due to Material Transformation (Transformation Beginning on Outside Diameter and Progressing Toward Center).

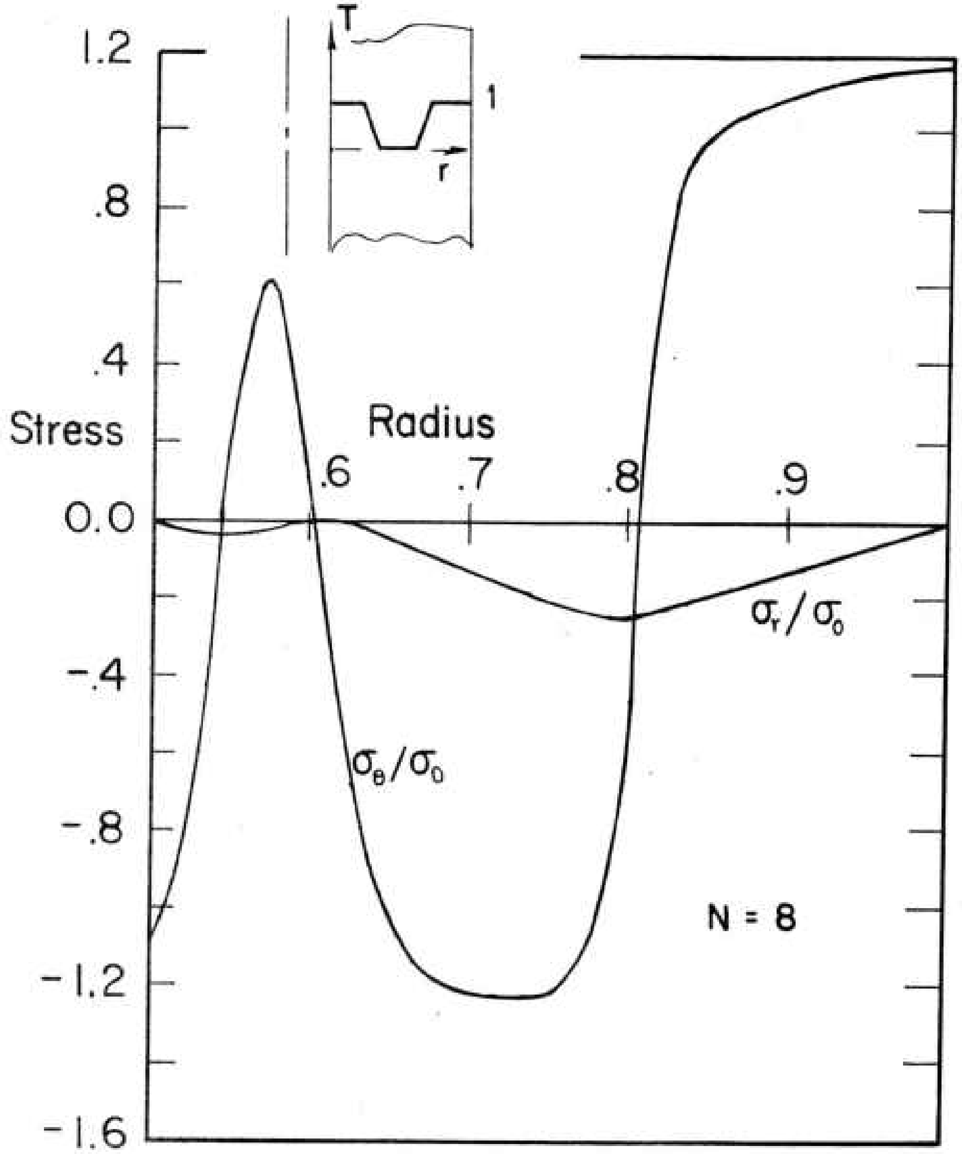

Figure 2. Residual Stresses in a Hollow Cylinder Due to Transformation (Transformation Occurring Symmetrically from the Inner and Outer Diameters).

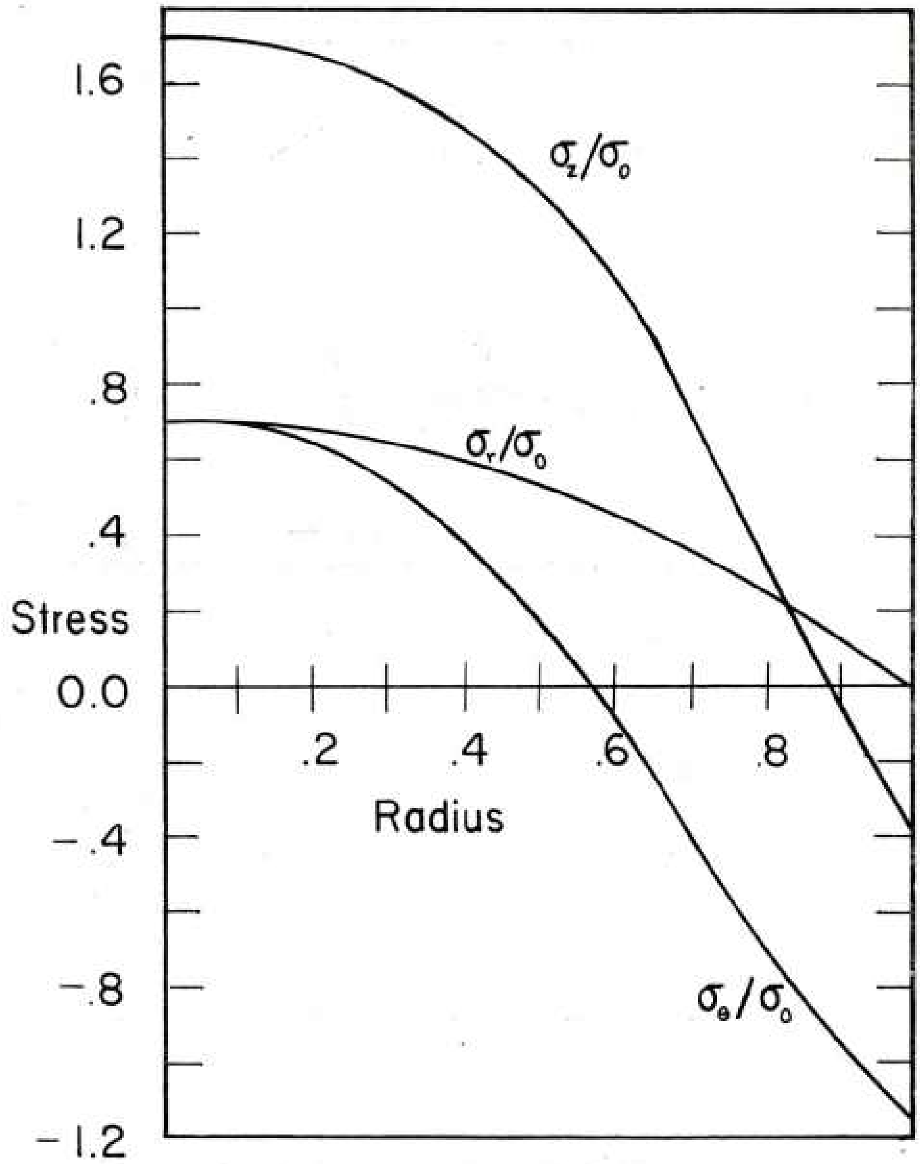

Figure 3. Residual Stresses in a Solid Cylinder Due to Quenching.

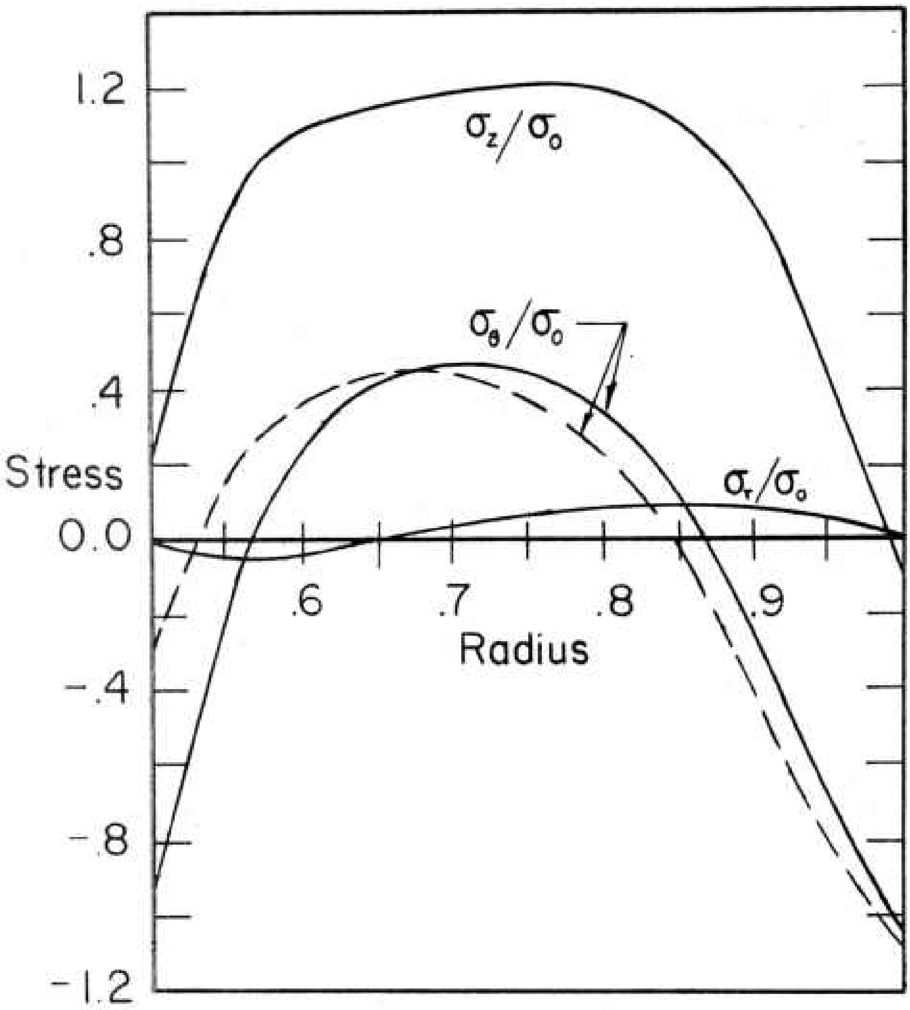

Figure 4. Residual Stresses in a Hollow Cylinder Due to Quenching.

Boundary Conditions:

h_1, h_2 = 12.2, ————————

h_1 = 6.1, h_2 = 12.2, ----------

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